Effect of disorder on critical short-time dynamics

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(Received 3 July 2001; revised manuscript received 3 October 2001; published 27 February 2002)

Critical short-time dynamics in a bond-diluted Ising model is investigated in this paper using numerical simulations. The effective static and dynamic critical exponents determined by the power-law scaling are found to depend strongly on bond concentration and initial state. For weak disorder, the short-time scaling relations for the system quenched from high temperature are observed to hold. In the strong dilution limit, multiscaling relations for the system starting from the ordered state are found. Corrections to the short-time scaling are proposed. The effect of disorder on critical short-time dynamics is discussed.

DOI: 10.1103/PhysRevE.65.036130 PACS number(s): 64.60.Ht, 75.10.Nr, 75.40.Mg

I. INTRODUCTION

Nonequilibrium critical dynamics has been an active research field for the past decades. The systems undergoing phase transitions usually show different stages of dynamic evolution after being quenched from high temperature to the critical temperature. Most well known is the late-time dynamics. In this regime, the order parameter decreases and shows power-law scaling behavior [1]. The scaling exponents are related to the static and dynamic critical exponents of the system. However, because of the diverse relaxation times at the critical temperature, the kinetics is difficult to characterize in real or infinite systems. Therefore critical exponents may not be easily obtained from direct dynamic scaling.

In recent years, many efforts have been made to develop dynamic scaling and universality for the critical dynamics in the *short-time regime*. It was first proven using the renormalization group method [2] that the order parameter shows ''initial slip'' behavior before it evolves into the late-time regime where it starts decreasing. A new critical exponent θ was introduced and the dynamic power-law scaling was obtained in short times [2,3]. The static and dynamic critical exponents determined by this short-time dynamic scaling are found to be consistent with those measured in equilibrium [3]. Since the measurement is carried out in short times, the off-equilibrium correlation length is still small compared to the system size. The accurate determination of critical exponents using critical dynamics is possible. Therefore, the short-time dynamic scaling provides an alternative approach for investigation of kinetics of the nonequilibrium systems.

Despite the initial success, many important issues pertinent to the short-time dynamic scaling still remain untouched in *disordered systems*. First, most of the disordered systems have ''glassy'' phases at low temperatures. Since the relaxation time is very large in this stage, it appears that the only way to characterize the phase transition in disordered system using nonequilibrium dynamics is to analyze its short-time behaviors at the transition temperature. However, whether or not the dynamic scaling holds in short-time regime in disordered systems still remains unknown. This question, therefore, needs to be studied in depth. Second, the concept of scaling and universality for phase transition in disordered systems is challenged in the on-equilibrium dynamics: The static and dynamic exponents are found to depend on the strength of disorder [4]. For example, the dynamic exponent z is quite different from that in the pure systems and varies with the strength of disorder. Because of its importance, this issue in a short-time off-equilibrium dynamics deserves further investigation. Third, the static and dynamic exponents obtained from short-time dynamic scaling for systems starting from two extreme initial states, i.e., completely disordered and ordered states, are found to be the same in many pure systems [3]. Whether the short-time dynamics in disordered system has the same property, or whether the critical exponents are independent of the initial state is still an open question.

Understanding of the nonequilibrium critical dynamics in disordered systems is important for the development and application of the short-time critical dynamics. Several recent works begin to address some of these issues in the critical short-time dynamics in disordered systems. For example, using renormalization group method [5] and Monte Carlo simulation [6,7], dynamic scaling is found to be valid in short times. However, the issue concerning the universality of static and dynamic exponents obtained from short-time dynamics was not addressed. Moreover, the effect of disorder on critical short-time dynamics and the correction to short-time scaling have not been systematically investigated. These and the aforementioned problems need to be resolved before the short-time critical dynamics can be applied to studying the phase transitions in disordered systems.

In this paper, an exactly solvable bond-diluted Ising model is employed to investigate the effect of disorder on the critical short-time dynamics. Monte Carlo simulation is used extensively in this paper. In the following section, we introduce a bond-diluted Ising model and critical short-time dynamics. In Sec. III, we analyze the critical short-time dynamics in systems starting separately from disordered and ordered initial states. Critical exponents obtained under these two conditions are compared. In Sec. IV, we calculate the dynamic exponent z for systems with various bond concentrations. Based on these results, we propose the corrections to critical short-time dynamics. In Sec. V, we analyze the effects of dilution and initial states on the critical short-time dynamics.

II. MODEL SYSTEM AND CRITICAL SHORT-TIME DYNAMICS

The McCoy-Wu model is an anisotropic Ising model that is solvable exactly [8,9]. The model used in this paper is a similar two-dimensional bond-diluted model on square lattice proposed by Longa [10]. Its Hamiltonian is written as

$$\hat{H}=-J_0\sum_{j,k} S_{j,k}S_{j,k+1}-\sum_{j,k} J_k S_{j,k}S_{j+1,k}, \tag{1}$$

where j labels the rows and k labels the columns of a square lattice. J_k denotes the strength of interaction among a column of spins and is a random variable with probability distribution function $P(J_k)=(1-p)\,\delta(J_k)+p\,\delta(J_k-J_0)$. The critical temperature T_c of the bond-diluted Ising model is given by the following expression:

$$\exp(-2pJ_0/k_BT_C)=\tanh(J_0/k_BT_C). \tag{2}$$

For bond concentration $0<p\leqslant 1$, long-range magnetic order exists and T_c is nontrivial.

According to critical short-time dynamics, after the system is quenched from an initial state with magnetization $m(t=0)=m_0\ll 1$, the global magnetization $m(t)$, the susceptibility $\chi(t)\propto M^2(t)$, and the autocorrelation function $C(t)$ should follow the power-law scaling relations [3]

$$M(t)\equiv\left[\left\langle\frac{1}{N}\sum_{i=1}^{N} S_i\right\rangle\right]\sim t^{\theta}, \tag{3a}$$

$$M^2(t)\equiv\left[\left\langle\left(\frac{1}{N}\sum_{2'=1}^{N} S_{2'}\right)^2\right\rangle\right]\sim t^{[d-2\beta/\nu]/z}, \tag{3b}$$

and

$$C(t)\equiv\left[\left\langle\frac{1}{N}\sum_{i=1}^{N} S_i(t)S_i(0)\right\rangle\right]\sim t^{-d/z+\theta}. \tag{3c}$$

Here $\langle\cdots\rangle$ denotes the average over spin configuration at $t=0$ (m_0 is fixed). $[\cdots]$ denotes the average over random-bond configurations for fixed p. $d=2$ is the spatial dimension. If the initial state is completely ordered ($m_0=1$), the magnetization and the Binder cumulant $U_1(t)$ should follow the power laws in time,

$$M(t)\sim t^{-\beta/\nu z} \tag{4a}$$

and

$$U_1(t)\equiv\frac{[\langle M^2(t)\rangle]}{[\langle M(t)\rangle]^2}-1\sim t^{d/z}. \tag{4b}$$

Equations (3) and (4) are valid in short times when the system size L is large.

In general, the critical short-time dynamic scaling for the kth moment of magnetization in a finite system can be written as

$$M^{(k)}(t,m_0;L)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{x_0}m_0;b^{-1}L), \tag{5a}$$

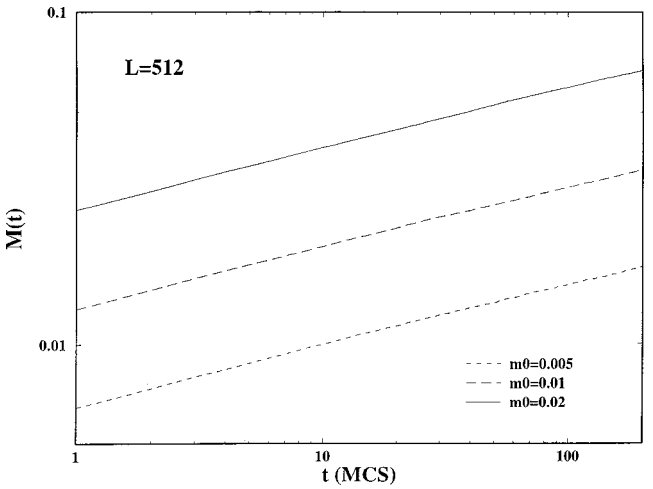

FIG. 1. Log-log plots of the total magnetization after the system is quenched from a disordered state. m_0 is the magnetization at $t=0$, $p=0.5$.

where L is the system size and b is a size-scaled factor, $x_0=\theta z+\beta/\nu$. The Binder cumulants can be written as [3]

$$U(t,L)=U(tb^z,bL), \tag{5b}$$

where $U(t)=U_1(t)$ for system with $m_0=1$ and $U(t)$ becomes $U_0(t)\equiv 1-[\langle M^4(t)\rangle]/3[\langle M^2(t)\rangle]^2$ for systems with $m_0=0$.

To verify the critical short-time dynamic scaling, we use heat-bath Monte Carlo simulation technique. An attempt of update of all spins is defined to be one Monte Carlo step (MCS). It was found [11] that the heat-bath spin update algorithm could reduce the microscopic time t_m (typically ~ 5 MCS) within which Eqs. (3) and (4) may not be valid. The periodic boundary conditions are used.

III. POWER-LAW SCALING FOR SHORT-TIME DYNAMICS IN SYSTEM WITH $p=0.5$

When $p=1.0$, the system governed by Eq. (1) is a pure Ising model and the critical dynamics has been extensively

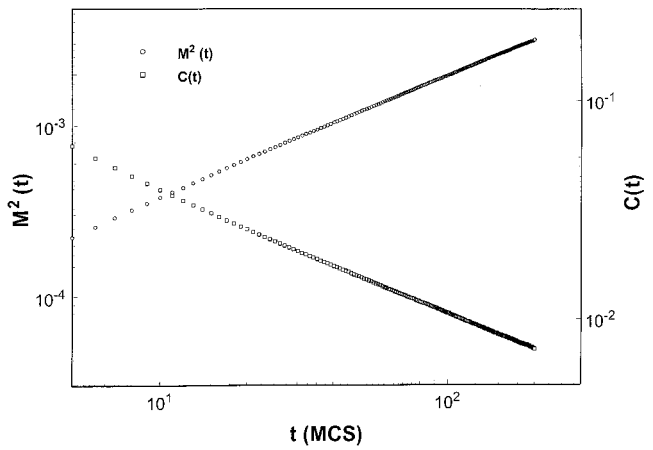

FIG. 2. Log-log plots of $M^2(t)$ and $C(t)$. $L=512$, $p=0.5$, and $m_0=0$.

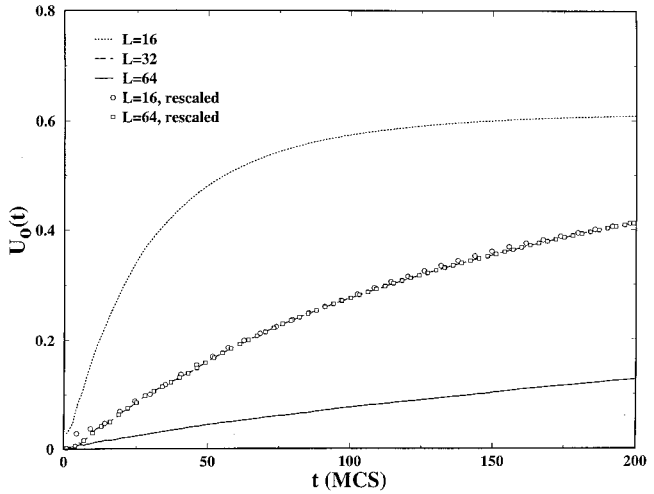

FIG. 3. Finite-size scaling for the Binder cumulant $U_0(t)$. The lines are original data and the symbols are the rescaled data. (The time is rescaled by a factor of 2^z.)

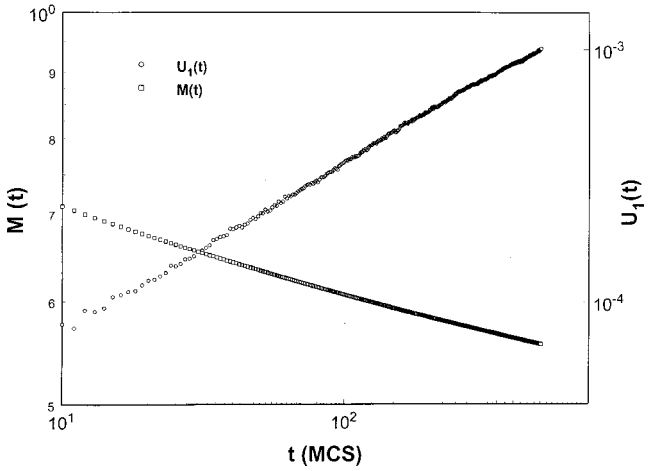

FIG. 4. Log-log plots of the magnetization $M(t)$ and the Binder cumulant $U_1(t)$. $L=512$, $p=0.5$, and $m_0=1$.

investigated [3]. When p tends to 0, power-law scaling [Eqs. (3)–(5)] for short-time dynamics needs to be checked. Therefore, we first consider a system with a median dilution at $p=0.5$. The critical temperature can be determined by Eq. (2) as $J_0/k_BT_c=0.609\,378$.

We investigated several systems with different sizes ($L=128$, 256, 512, and 1024). We find as $L\geqslant 256$, there is no finite-size effect on the physical quantities measured. In the system with $L=512$, the bond-configuration average and the initial spin-configuration average for each physical quantity are 5000 and 1000, respectively.

For each of the systems, we prepared two initial states. One is the ordered state with $m_0=1$; and another is the well-prepared disordered state with $m_0=0$. The dynamics are studied for each system following the subsequent evolution from each of the initial states.

Figure 1 shows the short-time increase of magnetization after the system is quenched from the disordered state. The data can be well fitted to Eq. (3a). The exponent is $\theta=0.176(3)$ if m_0 is extrapolated to zero. $M^2(t)$ and $C(t)$ of the system, which started from a completely disordered state ($m_0=0$), are shown in Fig. 2. They are all fitted well to power-law relations. The exponents in Eqs. (3b) and (3c) are $d/z-2\beta/z\nu=0.647(9)$ and $d/z-\theta=0.554(0)$, respectively. Finite-size scaling relations [Eqs. (5)] are also tested. The exponents $2\beta/\nu$ and z can be measured. Figure 3 shows the determination of dynamic exponent z by rescaling the Binder cumulant $U_0(t)$. In Table I, the exponents θ, β/ν, and z determined by power-law scaling and finite-size scaling relations are listed.

The evolution of magnetization $M(t)$ and Binder cumulant $U_1(t)$ after the system is released from an ordered state ($m_0=1$) are shown in Fig. 4. In the time regime (1–500 MCS) considered, $M(t)$ and $U_1(t)$ cannot be fitted well to power-law relations. If we take the microscopic time to be 150 MCS, $M(t)$ and $U_1(t)$ can be fitted to Eqs. (4a) and (4b), respectively, with acceptable errors. The exponents $2\beta/\nu$ and z are also determined by finite-size scaling for the second moment $M^{(2)}(t)$. Table I lists the results.

In Table I we compare d/z and $\beta/z\nu$ obtained from the short-time dynamics of the systems that start from the initial states with $m_0=1$ and $m_0=0$. The data listed in Table I indicate that the exponents determined from these two states are significantly different if the scaling relations [Eqs. (3)–(5)] are used. In particular, much large difference is observed for the exponent β/ν.

To summarize, we find that in a bond-diluted Ising model with $p=0.5$, the short-time power-law dynamic scaling is exact if the initial state is completely disordered ($m_0=0$). The exponents determined by power-law scaling and finite-size scaling relations are consistent with each other. However, when the initial state is ordered ($m_0=1$), the microscopic time t_m is found to be as large as 150 MCS. Most striking is the result where the static and dynamic exponents

TABLE I. The scaling exponents obtained from Eqs. (3)–(5) for a bond-diluted Ising model with bond concentration $p=0.5$.

	$m_0=1$		$m_0=0$		
	d/z	$\beta/z\nu$	θ	d/z	$\beta/z\nu$
Power-law scaling	0.617 ± 0.005	0.054 ± 0.002	0.176(2)	0.73(1)	0.042(0)
Finite-size scaling	0.71(8)	0.04(0)		0.78(5)	0.04(5)

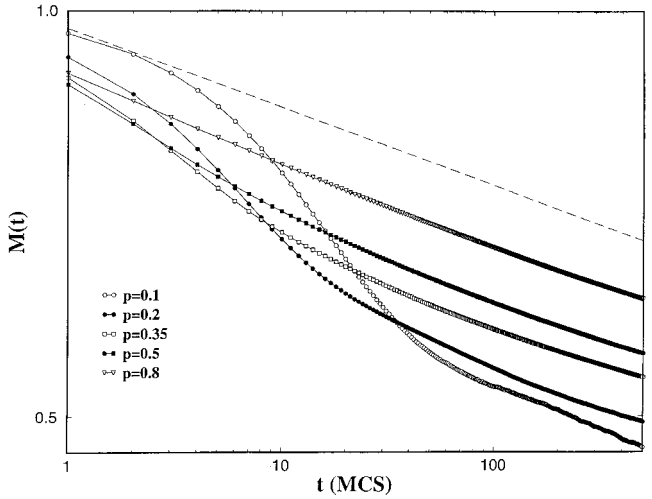

FIG. 5. Log-log plots of the magnetization $M(t)$ in the systems with different bond concentrations p. $m_0=1$, $L=512$. The dashed line is $M(t)$ for a pure Ising model.

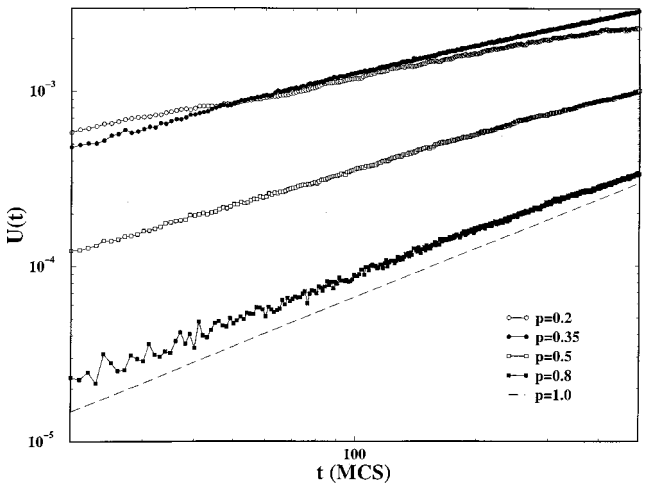

FIG. 6. Log-log plots of the Binder cumulants $U_1(t)$ in the systems with different bond concentrations p. $m_0=1$, $L=512$. The dashed line is $U_1(t)$ for a pure Ising model.

obtained from short-time dynamics with different initial states are not consistent with each other. This discrepancy will be considered below and the corrections to the scaling of the short-time dynamic in the system starting from ordered state will be addressed.

IV. EFFECT OF DISORDER ON THE CRITICAL SHORT-TIME DYNAMICS

A. Effect of disorder on critical exponents

We now investigate the critical short-time dynamics in the systems with different bond concentrations. At $t=0$, the systems are completely ordered ($m_0=1$). Figure 5 shows the effect of p on the evolution of magnetization $M(t)$ in early short-time regime ($t<1000$ MCS). When p decreases, $M(t)$ deviates significantly from the power-law scaling relation [Eq. (4a)]. If we define $\Delta(t)\equiv d[\ln M(t)]/d[\ln(t)]$, and the microscopic time t_m is assumed to be the crossover time when $\Delta(t)$ becomes a constant. Then $M(t)$ can be fitted to Eq. (4a) in time regime $t\geqslant t_m$, and the effective exponent is measured. Table II lists t_m and corresponding effective exponent $\beta/z\nu$ for the systems with various bond concentrations p. The Binder cumulants $U_1(t)$ are shown in Fig. 6. The effective exponent d/z can be determined by fitting the data in macroscopic time regime ($t\geqslant t_m$) to Eq. (4b). Table II lists all the results.

The effective exponents d/z and $\beta/z\nu$ as functions of p are plotted in Fig. 7(a). It shows that $\beta/z\nu$ is independent of p and d/z can be fitted to a linear function of p under the assumption that the power-law scaling relations [Eqs. (4)] hold in early short-time regime.

The exponents β/ν and z can also be measured by finite-size scaling relations [Eqs. (5)] in the system starting from completely disordered state. Table II lists z and β/ν for different systems. Figure 7(b) shows the dynamic exponents determined from systems starting from disordered and ordered states, respectively.

B. Correction to short-time critical scaling

From the Monte Carlo studies, we observed the difference in short-time critical scaling in a bond-diluted Ising model. As mentioned earlier, when the system is quenched from high-temperature disordered state, the short-time scaling is exact and the critical exponents do not change with bond concentrations significantly. But for the system starting from the ordered state, the power-law scaling relations are completely broken down in short times. This discrepancy is very interesting and also important for critical short-time dynamics. As the marginal irrelevance of disorder is expected in disordered system with little dilution [13,14], we can make necessary corrections to the short-time dynamic scaling for pure systems [Eqs. (3)–(5)] and solve the problem in the bond-diluted Ising model.

TABLE II. The exponents obtained from short-time scaling. The effective exponents d/z and $\beta/z\nu$ are determined by power-law scaling for systems with $m_0=1$ ($L=512$). The exponent z and β/ν are determined by finite-size scaling for a pair of lattices $L=32$ and $L=64$, $m_0=0$.

p	0.1	0.2	0.35	0.5	0.8	1.0
t_m (MCS)	5000	507	98	51	32	2
d/z		0.396(3)	0.502(8)	0.616(9)	0.79(6)	0.92(5)
$\beta/z\nu$	0.05(9)	0.05(4)	0.054(6)	0.053(9)	0.054(6)	0.056(8)
z	2.48	2.54	2.52	2.58	2.36	2.14(5)
β/ν	0.115	0.115	0.124	0.115	0.120	0.125

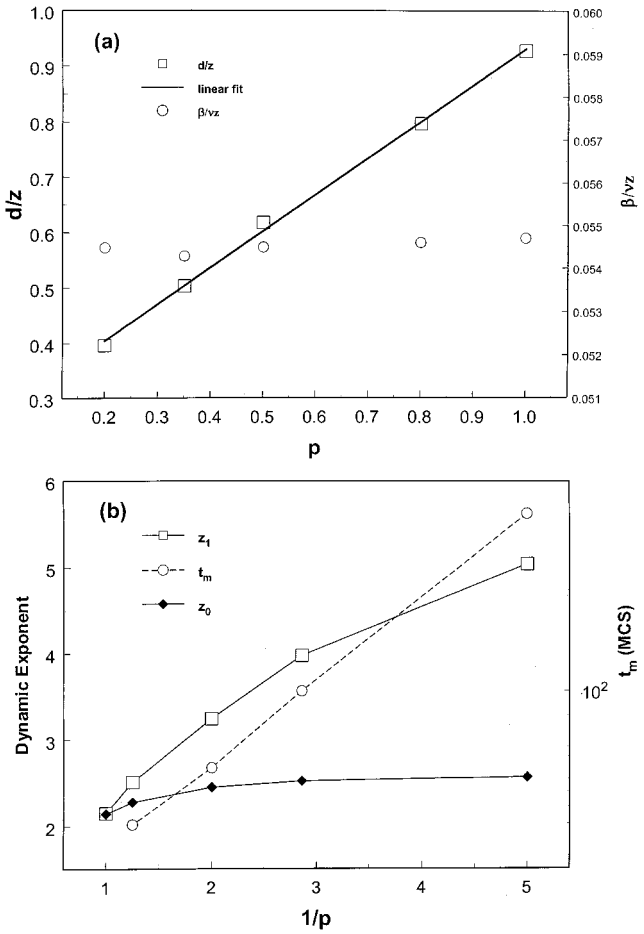

FIG. 7. (a) The exponents d/z and β/ν at different bond concentrations p. $m_0=1$ and $L=512$. The solid line is a linear fit. (b) The dynamical exponents and crossover time t_m at various bond concentrations p. z_1 is the dynamical exponent obtained from dynamical scaling for $U_1(t)$ and z_0 is the dynamical exponent obtained by finite-size scaling for $U_0(t)$. The right axis is in logarithm scale. Lines are guides to the eye.

Corrections to short-time critical scaling have been studied in some disordered systems and systems with metastable states [15–18]. Here we look at the general correction to the critical dynamics in the bond-dilute McCoy-Wu model.

Figure 8 shows the evolution of magnetization up to 10 000 MCS after a completely ordered system is placed at the critical temperature. The deviation from power law is obvious when p tends to zero. We now consider the leading scaling correction for systems with various bond concentrations. From the critical singularity of specific heat in McCoy-Wu model, the correlation length $\xi(t)$ at the critical

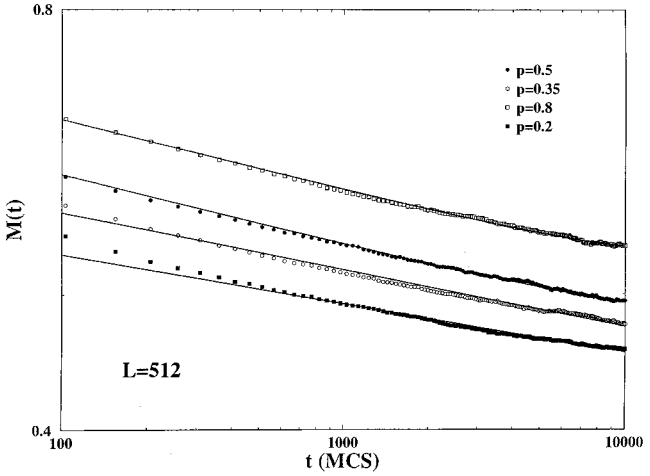

FIG. 8. Log-log plots of the magnetization (symbols) in the time regime [100, 10 000] in systems with different bond concentrations p. $m_0=1$, $L=512$. The solid lines are the fits of Eq. (7).

temperature can be proposed as [8,9]

$$\xi(t)\sim t^{1/z}/[1+C_0\ln(t)]^{1/z'}, \tag{6}$$

where $z'(p)$ is a new exponent and is a function of p. When $p=1$, $1/z'(p)=0$ for a pure Ising model. According to Eq. (6), Eqs. (3)–(5) can be modified. The correction to Eq. (4a) can be written as

$$M(t)\sim t^{-\beta/\nu z}/[1+A(p)\ln(t)]^{B(p)}, \tag{7}$$

where $A(p)$ and $B(p)$ are functions that depend only on the bond concentration. Assume that $z'(p)=z$ is independent of p when p tends to zero, then Eq. (5b) can be changed to

$$U(t,L)=U(tb^z/[1+A(p)\ln t],bL). \tag{8}$$

If $A(p)$ is small, Eqs. (7) and (8) are consistent with power-law scaling relations in the early short-time regime, i.e., $t_m<t<1000$ MCS.

Corrections to $M(t)$ in systems starting from $m_0=1$ are shown in Fig. 8 ($\beta/z\nu$ is fixed at 0.0570). Table III lists the corresponding $A(p)$ and $B(p)$. Binder cumulants in systems starting from $m_0=0$ are rescaled by Eq. (8). Figure 9 shows the finite-size scaling for $U_0(t)$ in systems with $p=0.35$. Two pairs of lattices, $(L_1,L_2)=(16,32)$ and $(L_1,L_2)=(32,64)$, are used to determine the exponent z. Table III lists all the results.

From the data in Table III and Figs. 8 and 9, we can devise proper corrections to scaling for short-time critical

TABLE III. The coefficients $A(p)$ and $B(p)$ of fits in Eq. (7). z is the dynamical exponent determined by Eq. (8) for system with $m_0=0$.

p	0.2	0.35	0.5	0.8
$A(p)$	0.120	0.062	0.098	0.10
$B(p)$	0.032	0.030	0.031	0.026
z	2.20	2.18	2.19	2.14

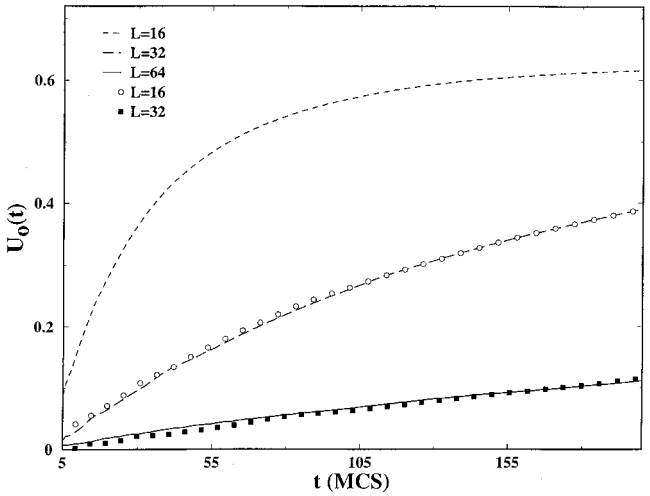

FIG. 9. Finite-size scaling for the Binder cumulant $U_0(t)$. The lines are original data and the symbols are the rescaled data. (The time is rescaled as $t/[1+A(p)\ln t]L^z$.) $p=0.35$ and $m_0=0$.

dynamics. The corrections to the scaling can be fitted very well to the Monte Carlo simulation results. The critical exponents determined from systems with $m_0=1$ and $m_0=0$ from this fitting are consistent with each other. It should be mentioned that several other corrections to short-time dynamics in the diluted Ising models are proposed recently, including the logarithm correction [15] and inverse power-law correction [16]. We have tested these correction forms. However, we find that they do not fit well the short-time dynamics in the bond-diluted Ising model.

V. DISCUSSION AND CONCLUSION

The results from the early works show that in disordered systems, the critical exponents depend on the strength of disorder [12]. In the bond-diluted Ising model governed by Eq. (1), it is found by the equilibrium critical scaling that the exponents z and β/ν increase with decreasing bond concentration p, and $z\to\infty$ as p tends to zero. The effective exponents z and β/ν obtained from power-law scaling in early short-time regime support these results quantitatively. However the effective static and dynamic exponents obtained by power-law short-time scaling for systems starting from ordered ($m_0=1$) and disordered ($m_0=0$) states are not consistent with each other.

For the system starting from a completely disordered state ($m_0=0$), the strength of disorder only affects the microscopic time t_m. It seems that t_m does not depend strongly on the bond concentration p. In the weak dilution limit, the power-law scaling relations [Eqs. (3)] may still be valid in short times. In the strong dilution limit ($p\to 0$), we found that the short-time dynamic scaling relations for the system starting from ordered states ($m_0=1$) are completely broken down. There exist three stages of dynamic evolution in the system starting from the ordered state. In time period of 100–1000 MCS, the power-law decay of $M(t)$ is observed, which is similar to that in a pure Ising system. Eq. (4a) is irrelevant to the disorder and the effective exponent $\beta/z\nu$ is independent of bond concentration. For strong bond dilution and in relatively longer times ($t>1000$ MCS), the effect of disorder on dynamics needs to be taken into account. The corrections to dynamic scaling result in consistent exponents, i.e., exponents are independent of disorder and initial states and the exponents are comparable with those of pure Ising model. In the microscopic time regime, the dynamic behavior of $M(t)$ is unique. We suspect that the initial microscopic kinetics and disorder in the system may be the cause for the behaviors in this regime. More detailed investigation is underway in a very strong dilution limit, for example, at $p=0.01$.

Why there exists the discrepancy between the short-time dynamics in systems with $m_0=0$ and $m_0=1$ could be attributed to the following reasons. In pure Ising systems, the short-range order can be achieved for both systems evolved from ordered and disordered states. But in bond-diluted Ising systems, because of the disorder, the correlation in the system starting from an ordered state is destroyed in short times. Therefore the initial state significantly affects the critical exponents determined by short-time dynamics in the disordered system.

In summary, we studied the critical short-time dynamics of the bond-diluted Ising models using Monte Carlo simulations. We found that when the dilution is large, the effective static and dynamic exponents determined from power-law scaling in early short-time regime in systems starting from disordered and ordered initial states are apparently different. General corrections to short-time dynamics are proposed and are found to fit the simulation results well.

ACKNOWLEDGMENTS

We gratefully acknowledge the support of this work provided by the U.S. Department of Energy (Grant No. DEFG02-99ER45784) and the Whiting School of Engineering at The Johns Hopkins University (M.L.).

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